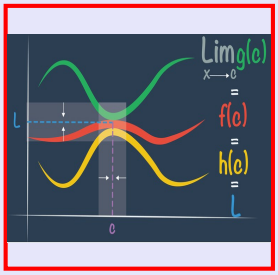
Math 261

Spring 2023

Lecture 44



Feb 19-8:47 AM

Theorem:

If $f(x)$ is continuous on $[a,b]$, then $f(x)$ is integrable on $[a,b]$ and $\int_a^b f(x)dx$ exists.

If $f(x)$ is integrable on $[a,b]$, then

$$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\cdot\Delta x$$

Riemann Sum

where $\Delta x = \frac{b-a}{n}$, and $x_i = a + i\Delta x$

$$\int_a^b f'(x)dx = f(x)\Big]_a^b$$ Definite integral

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C \quad \text{if } n \neq -1.$$

May 3-8:50 AM

Evaluate $\int_0^2 x^2\,dx$ by Riemann Sum.

$a=0$, $b=2$, $\Delta x = \frac{b-a}{n} = \frac{2}{n}$

$x_i = a + i\,\Delta x = 0 + i\cdot\frac{2}{n} \Rightarrow x_i = \frac{2i}{n}$

$f(x) = x^2 \qquad f(x_i) = \left(\frac{2i}{n}\right)^2 = \frac{4}{n^2} i^2$

$f(x_i)\cdot\Delta x = \frac{4}{n^2} i^2 \cdot \frac{2}{n} = \frac{8}{n^3} i^2$

$\sum_{i=1}^{n} f(x_i)\cdot\Delta x = \sum_{i=1}^{n} \frac{8}{n^3} i^2 = \frac{8}{n^3}\sum_{i=1}^{n} i^2 = \frac{8}{n^3}\cdot\frac{n(n+1)(2n+1)}{6}$

$= \frac{16n^3 + \cdots}{6n^3}$

$\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\cdot\Delta x = \lim_{n\to\infty} \frac{16n^3 + \text{Junk}}{6n^3} = \frac{16}{6} = \boxed{\frac{8}{3}}$

$\int_0^2 x^2\,dx = \boxed{\frac{8}{3}}$

Using Power Rule

$\int_0^2 x^2\,dx = \frac{x^3}{3}\Big]_0^2 = \frac{2^3}{3} - \frac{0^3}{3} = \boxed{\frac{8}{3}}$

May 3-8:56 AM

Evaluate $\int_0^2 (x^2 - 2x + 3)\,dx$

$= \left(\frac{x^3}{3} - \cancel{2}\cdot\frac{x^2}{\cancel{2}} + 3x\right)\Big]_0^2$

$= \left[\frac{2^3}{3} - 2^2 + 3(2)\right] - \left[\cancel{\frac{0^3}{3} - 0^2 + 3(0)}\right] \to 0$

$= \frac{8}{3} - 4 + 6 = \frac{8}{3} + 2 = \boxed{\frac{14}{3}}$

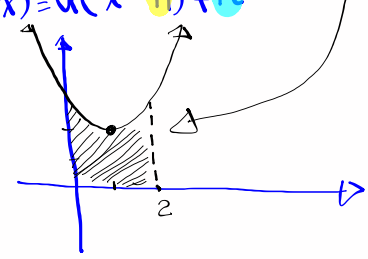
$f(x) = x^2 - 2x + 3 = x^2 - 2x + 1 + 2 = (x-1)^2 + 2$

$f(x) = (x-1)^2 + 2 \qquad f(x) = a(x-h)^2 + k$

Parabola

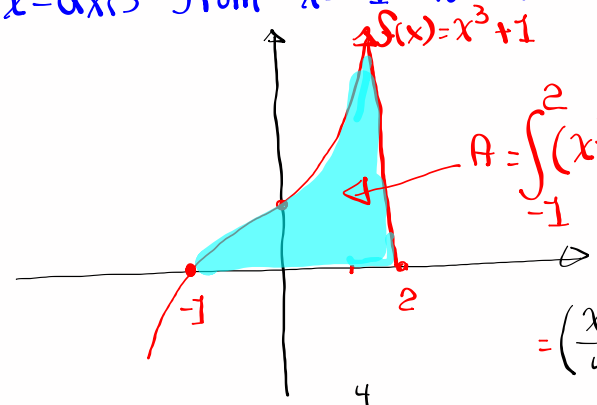
opens up

Vertex $(1, 2)$



May 3-9:04 AM

Find the area below $f(x)=x^3+1$, above x-axis from $x=-1$ to $x=2$.



$$A=\int_{-1}^{2}(x^3+1)\,dx$$

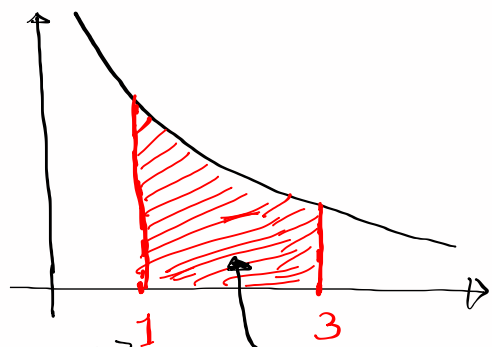
$$=\left(\frac{x^4}{4}+x\right)\Big]_{-1}^{2}$$

$$=\left(\frac{2^4}{4}+2\right)-\left(\frac{(-1)^4}{4}-1\right)=\boxed{\frac{16}{4}}+2\boxed{-\frac{1}{4}}+1$$

$$=\frac{15}{4}+3=\frac{15}{4}+\frac{12}{4}=\boxed{\frac{27}{4}}$$

May 3-9:11 AM

Find the area below $f(x)=\frac{1}{x^2}$, above x-axis, on $[1,3]$.



$f(x)=\frac{1}{x^2}$ is cont. on $[1,3]$

$f(x)>0$ on $[1,3]$

$$A=\int_1^3\frac{1}{x^2}\,dx=\int_1^3 x^{-2}\,dx=\frac{x^{-2+1}}{-2+1}\Big]_1^3$$

$$=\frac{x^{-1}}{-1}\Big]_1^3=\frac{-1}{x}\Big]_1^3=-\left(\frac{1}{3}-\frac{1}{1}\right)=-\left(\frac{1}{3}-1\right)=-\left(\frac{-2}{3}\right)$$

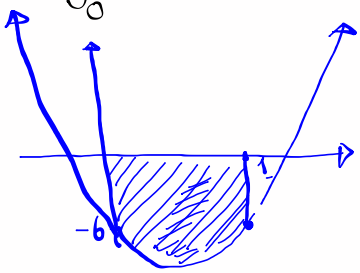
$$=\boxed{\frac{2}{3}}$$

May 3-9:18 AM

Evaluate $\int_0^1 (x+2)(x-3)\,dx$

$$= \int_0^1 (x^2 - 3x + 2x - 6)\,dx$$

$$= \int_0^1 [x^2 - x - 6]\,dx = \left(\frac{x^3}{3} - \frac{x^2}{2} - 6x\right)\Big]_0^1$$



$$= \left(\frac{1}{3} - \frac{1}{2} - 6\right) - (\cancel{0 - 0 - 0} \to 0)$$

$$= \frac{2 - 3 - 36}{6} = \boxed{\frac{-37}{6}}$$

is below x-axis.

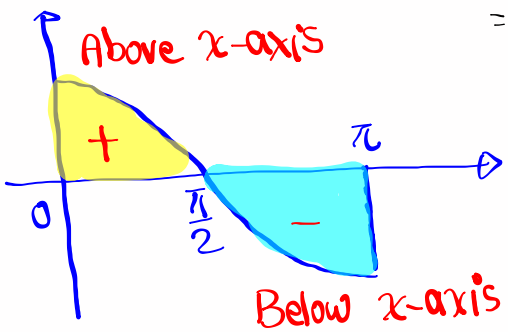
That is why

May 3-9:24 AM

Evaluate $\int_0^{\pi} \cos x\,dx = \sin x\Big]_0^{\pi}$

$$= \sin \pi - \sin 0$$

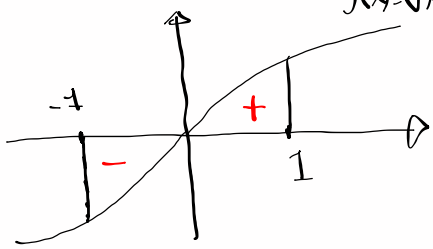
$$= 0 - 0 = \boxed{0}$$



May 3-9:31 AM

Evaluate $\int_{-1}^{1} \sqrt[3]{x}\, dx = \int_{-1}^{1} x^{1/3} dx$

$f(x) = \sqrt[3]{x}$



$$= \left.\frac{x^{\frac{1}{3}+1}}{\frac{1}{3}+1}\right]_{-1}^{1} = \left.\frac{x^{\frac{4}{3}}}{\frac{4}{3}}\right]_{-1}^{1}$$

$$= \left.\frac{3}{4}\sqrt[3]{x^4}\right]_{-1}^{1}$$

$$= \frac{3}{4}\left(\sqrt[3]{1^4} - \sqrt[3]{(-1)^4}\right) =$$

$$= \frac{3}{4}\left(\sqrt[3]{1} - \sqrt[3]{1}\right) = \boxed{0}$$

May 3-9:36 AM

Evaluate $\int_{1}^{2} \left(x + \frac{1}{x}\right)^2 dx$

$f(x) = \left(x + \frac{1}{x}\right)^2$ is cont. on $[1,2]$

$$= x^2 + 2 \cdot x \cdot \frac{1}{x} + \frac{1}{x^2} = x^2 + 2 + x^{-2}$$

$$\int_{1}^{2} \left(x + \frac{1}{x}\right)^2 dx = \int_{1}^{2} \left(x^2 + 2 + x^{-2}\right) dx =$$

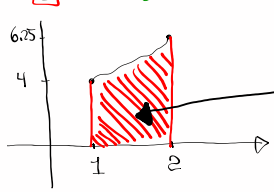
$$= \left.\left(\frac{x^3}{3} + 2x + \frac{x^{-1}}{-1}\right)\right]_{1}^{2}$$

$$= \left.\left(\frac{1}{3}x^3 + 2x - \frac{1}{x}\right)\right]_{1}^{2}$$

$$= \left(\frac{1}{3} \cdot 2^3 + 2(2) - \frac{1}{2}\right) - \left(\frac{1}{3} \cdot 1^3 + 2(1) - \frac{1}{1}\right)$$

$$= \boxed{\frac{8}{3}} + 4 - \frac{1}{2} - \boxed{\frac{1}{3}} - 2 + 1 = \frac{7}{3} + 2 + \frac{1}{2}$$

$$= \frac{14 + 12 + 3}{6} = \boxed{\frac{29}{6}}$$



May 3-9:41 AM